VCST Internal Memo

Title: Assessment of FP-11' Polarization Sensitivity for the JPSS J1 VIIRS VisNIR Bands and Comparison to FP-11 Measurements
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Author: Jeff McIntire
To: Xiaoxiong Xiong and James Butler
Cc: Hassan Oudrari, Kwo-Fu (Vincent) Chiang, Jon Fulbright and Aisheng Wu

References

[1] TP1544640-2222 Rev A, 'Polarization Sensitivity FP-11 Test Procedure,' Raytheon.
[2] VCST_TECH_REPORT_14_083, 'Preliminary Analysis of Polarization Sensitivity Retest Data,' Jeff McIntire, November 24, 2014.
[3] 'JPSS J1: Results from FP-11 Polarization Post TVAC,' Raytheon, December 5, 2014.
[4] VCST_TECH_MEMO_14_002, 'Assessment of FP-11 Polarization Sensitivity for the VIIRS F2 VisNIR Bands,' Jeff McIntire, March 13, 2014.
[5] VIIRS PRD Rev C, 'Joint Polar-orbiting Satellite System (JPSS) Visible Infrared Imaging Radiometer Suite (VIIRS) Performance Requirements Document (PRD).'
[6] VIIRS.10.15.036, 'Polarization Test Source Assembly (PTSA) Calibration and Performance Verification,' Josh Cohen, Dan Tiffany, Eslim Monroy, and Tung Wang, November 13, 2013.
[7] Mathematical Methods in the Physical Sciences, Mary L. Boas, John Wiley & Sons, 1985.

1. Introduction

JPSS J1 VIIRS sensor polarization sensitivity was measured for the VisNIR bands during FP-11' in post-TVAC testing [1]. Preliminary NASA analysis was reported in [2] and Raytheon reported their results in [3]. This work will provide an overview of the test setup and objectives, analysis methodology, and results as well as a comparison to earlier FP-11 measurements [4].

2. Objective

FP-11 measurements were performed at a series of 7 discrete scan angles. An interpolation scheme is used to determine the polarization sensitivity between the measured angles. The objective of the FP-11' test was to confirm that the scan angle interpolation of FP-11 results was valid. In addition, the FP-11' test was also used to independently determine the sensitivity of the VIIRS instrument to input linearly polarized light. There are two specifications which relate to the polarization sensitivity of VIIRS [5]:

> **V_PRD-12624** – The VIIRS Sensor linear polarization sensitivity of the VIS and NIR bands shall be less than or equal to the values indicated in Table 1 for scan angles less than 45 degrees off Nadir.
>
> **V_PRD-12667** – The VIIRS Sensor linear polarization sensitivity shall be measured within a characterization uncertainty of 0.5% (one sigma) for scan angles less than 55.84 degrees off Nadir.

Only the first specification will be addressed in this memo; the second will be addressed in a subsequent memo.

2. Test Equipment

The Polarization Test Source Assembly (PTSA) was used to measure the polarization sensitivity of the JPSS J1 VIIRS sensor [1]. This setup consisted of an integrating sphere, polarizer sheets, shaping filters, and various baffling [6]; the setup was nearly identical to FP-11 measurements. VIIRS was placed on a rotating mount such that the angle at which VIIRS viewed the PTSA could be varied. The PTSA was viewed at the following VIIRS scan angles: -37, -30, -15, -8, and 4 degrees. A general schematic of the test setup is shown in Figure 1 indicating the placement of the filters and polarizers relative to the path. Figure 2 references the polarizer angle to VIIRS coordinates system including the rotation of the polarizer (in this figure, the view is from VIIRS through the polarizer to the integrating sphere).

An integrating sphere (SIS-100-2) was used as a source for FP-11' testing. The SIS-100-2 is a 100 cm integrating sphere with a 12 inch circular aperture. The spectral output of the SIS-100-2 is shown in Figure 3 for two different lamp levels: ABCDEFG and ABCDFGH (not shown) were used with the Sonoma filter and BDF was used without any shaping filter (see below for the filter description). During testing, the ABCDEFG configuration proved unstable and ABCDFGH was substituted. The effected data (highlighted in red in Table 2 and 3) was not used in this analysis and was recollected.

The polarization sheet used during testing was the BVONIR. This polarizer was mounted on a rotary stage which could be cycled from 0 and 360 degrees. A second fixed polarizer of the same type was used in conjunction with the first to measure the polarizer efficiency. The degree of linear polarization for the BVONIR polarizer is plotted in Figure 4.

The shaping filter used during testing was the Sonoma filter. The Sonoma filter is a long wave blocking filter with very low transmittance above about 650 nm. The transmittance of the shaping filter is plotted in Figure 5. Figure 6 shows the source spectra for all the test configurations measured in FP-11' with the BVONIR polarizer in comparison to the TOA spectra.

In addition, a lollipop obscuration was inserted into the optical path (preceding the polarizer, but after the Sonoma filter) to investigate possible stray light contamination.

3. Test Configuration

Two test configurations were used during FP-11' testing: BVONIR with the Sonoma filter and BVONIR without the Sonoma filter. Stray light, efficiency, and polarization sensitivity tests were performed for each configuration.

First, a stray light investigation was performed for each configuration by performing two tests: one with the source off and the room dark, and the other with the source on and a lollipop obscuration inserted into the path. The rotating polarizer was then cycled through 25 different angles from 0 to 360 degrees in 15 degree increments for both tests.

Next, a second fixed polarizer was inserted into the path between the rotating polarizer and VIIRS for both configurations. The rotating polarizer was then cycled through 25 different angles from 0 to 360 degrees in 15 degree increments.

Lastly, the polarization sensitivity of VIIRS instrument was measured for both configurations. The BVONIR configurations were measured at all 5 scan angles including three repeated measurements at

-8 degrees. The configurations with the Sonoma filter were used to measure the sensitivity of bands M1 – M3, while the sensitivity of bands I1 – I2 and M4 – M7 was measured without the Sonoma filter. However, useful data was collected for some bands in both configurations (with and without the Sonoma filter).

4. Methodology

A standard Fourier analysis was used to determine the polarization sensitivity of the VIIRS instrument [7]. The Fourier expansion is written as

$$dn = \frac{1}{2}c_0 + \sum_n c_n \cos(n\alpha) + \sum_n d_n \sin(n\alpha), \quad (1)$$

where the Fourier coefficients are defined by the following:

$$c_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos(n\theta)\, dn(\theta)\,\partial\theta, \quad (2)$$

and

$$d_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin(n\theta)\, dn(\theta)\,\partial\theta. \quad (3)$$

In this work, only the zeroth through fourth order Fourier coefficients were calculated (for the polarizer efficiency tests, only the zeroth and second order terms were calculated). Eq. (1) can be rewritten as

$$dn = \frac{1}{2}c_0\left[1 + \sum_{n=1}^{4} a_n \cos(n\alpha - \delta_n)\right], \quad (4)$$

where the linear polarization sensitivity of the instrument (referred to below as the polarization factor) is defined as

$$a_n = \frac{\sqrt{c_n^2 + d_n^2}}{\frac{1}{2}c_0\sqrt{a_2^{eff}}} \quad (5)$$

and the phase angle is defined as

$$\delta_n = \tan^{-1}\left(\frac{d_n}{c_n}\right). \quad (6)$$

The efficiency of the polarizer is defined as

$$a_2^{eff} = \frac{\sqrt{c_2^2 + d_2^2}}{\frac{1}{2}c_0}, \quad (7)$$

and was determined from the cross polarizer testing.

5. Analysis

The data analyzed during the FP-11’ testing is listed in Tables 2 and 3 (including the type of test, number of collects, scan angle, and samples used). In this work, 30 EV samples were used in the processing. The sample window was chosen to minimize the stray light observed in the lollipop obscuration test. The OBC BB view data was used as a dark reference, averaged per scan, and subtracted from each EV pixel of the corresponding scan. Then all EV pixels were first averaged over a particular scan and then all scans were averaged in each collect (a 3-sigma outlier rejection was used in each average). Note that the BB data was first truncated from 14 to 12 bits in order to remove any bias between the EV and BB data.

Each collect corresponded to a measurement at a discrete polarizer sheet angle. The angle was cycled from 0 to 360 degrees in 15 degree increments. This set of 25 measurements was then used to determine the Fourier coefficients in Eqs. (2) and (3), and finally the linear polarization sensitivity and phase from Eqs. (5) and (6). The cross polarizer efficiency test was analyzed first; then the efficiency correction defined by Eq. (7) was used in the final polarization sensitivity calculation.

FP-11 Fourier coefficients [4] were used to linearly interpolate from the measured FP-11 scan angles (-55, -45, -20, -8, 22, 45, and 55) to the scan angles measured in FP-11' (-37, -30, -15, and 4). These interpolated coefficients were then used to construct the polarization factors and phases from Eqs. (5) and (6). The interpolated polarization factors and phases were then compared to FP-11' measurements.

6. Results

Data from the stray light tests was analyzed for both configurations for the dark and lollipop tests. The stray light observed for all detectors in band M7 is shown in Figures 7 (dark) and 8 (lollipop) using the BVONIR polarizer without the Sonoma filter. The stray light observed in the dark configuration is consistent with zero. Some small positive signal was recorded using the lollipop configuration (less than ~0.2 dn for all detectors). All other bands showed similar results (all detectors less than ~0.2 dn). Ideally, any stray light would constitute a pedestal that would need to be subtracted from the polarization sensitivity measurements; however, given the low levels of stray light observed, no additional processing was conducted.

Cross polarizer test data was analyzed for both configurations and the DoLP of the polarizers in each configuration was determined. The efficiency was calculated for each band, detector, and HAM side; the band average results for the BVONIR with and without the Sonoma filter are shown in Figure 9 (also averaged over HAM side). Note that there is good agreement between the measurements for M1 – M3; the signal is much lower for these bands without the Sonoma filter, and so the efficiency is less well determined. Also, the Sonoma filter blocks only part of the in-band I1 radiance; as a result, the efficiencies for the different configurations are not consistent. For the remaining bands, the signal with the Sonoma filter was too low to reliably determine the DoLP. Figure 9 also compares the FP-11 and FP-11' measurements for both configurations; the FP-11 results are consistently larger (~0.3 %) for all bands.

The polarization sensitivity was derived for all VisNIR bands, detectors, HAM sides, and five scan angles. Figure 10 plots the measured dn for band M7, HAM side 0 using the BVONIR polarizer without the Sonoma filter at a scan angle of -8 degrees (UAID 4304499). The lines indicate the calculated Fourier series using the zeroth through fourth order terms; the Fourier series reproduces the observed behavior very well. Note that the amplitude and phase of the Fourier series varies with detector. Figure 11 shows the zeroth through fourth order Fourier coefficients for M7, HAM side 0 versus scan angle for all detectors. Only the zeroth and second order terms show non-negligible results (note the larger offset observed for some data resulted from using the ABCDFGH lamp configuration instead of ABCDEFG). The results shown in Figures 10 and 11 are indicative of all VisNIR bands. Figure 12 compares the second order term a_2 across repeated measurements at a scan angle of -8 degrees (band M1, HAM side 0, with Sonoma filter) using data from both FP-11 and FP-11'. The repeated measurements at -8 degrees generally agree well for each BVONIR configuration. The results shown in Figure 12 are representative of all VisNIR bands; the maximum difference between the two test runs (over bands, detectors, and HAM sides) is less than 0.08 %.

The final polarization factors (a_2) are shown for all bands, detectors, and scan angles in Figure 13 (14) for HAM side 0 (1). The maximum values per band and scan angle are shown in Table 4 (5) for HAM side 0 (1). The final polarization results were derived using the BVONIR with the Sonoma filter for M1 – M3 and without the Sonoma filter for I1 – I2 and M4 – M7. The first specification in Section 2 applies directly to a_2 for scan angles less than 45 degrees. For some scan angle – detector combinations, bands M1 – M4 are non-compliant with the specification (up to ~ 6.4 % for M1, ~4.3 % for M2, ~3.0 % for M3, and ~4.3 % for M4). There is also noticeable HAM side dependence with up to about 1.0 % difference, where the HAM side 1 results are generally larger. Also, note that there is considerable variation both with detector and with scan angle (greater than 4 % for M1 at -55 degrees scan angle). The SNPP VIIRS a_2 are also listed in Tables 7 and 8 for comparison [10]; note that for almost all cases, J1 has larger polarization sensitivity than SNPP. The corresponding final polarization phases (δ_2) are shown for all bands, detectors, and scan angles in Figure 15 (16) for HAM side 0 (1). The phase determination becomes difficult when a_2 is small, as a small change in the c_2 or d_2 causes a large change in δ_2 when c_2/d_2 approaches 0 or 1.

The difference between FP-11' measurements and results determined by linearly interpolating FP-11 measurements to the FP-11' scan angles are listed in Tables 6 and 7 for the polarization factor and phase. The differences shown are the maximum of HAM sides and detectors per band. The polarization factors are consistent to within 0.11 %. The phase angle differences for bands I1 and M1 – M4 are less than 4.5 degrees. In contrast, the phase angle differences for bands I2 and M5 – M7 are as large as 22 degrees; this is the result of the phase shifting for some detectors from 0 to 2π. The linear interpolation is conducted on Fourier coefficients c_2 and d_2 independently and then the a_2 and δ_2 were calculated; if c_2 approaches zero in a particular interval, then the phase angle becomes less well determined [see Eq. (6)]. As an example, Figure 17 plots the M5 phase angles per detector for HAM 0 (both FP-11 and FP-11' measurements). Note that the phase shift occurs for the higher number detectors, affecting the interpolation for scan angles -37 and -30, whereas the differences in Table 7 are smaller for scan angle -15 and 4. Figure 18 shows a similar plot for M7 HAM 0 (indicated the cause of the lower agreement at scan angles -15 and 4). Note that such phase angle shift occur where the polarization factor approaches zero. The additional scan angles measured during FP-11', when used in concert with FP-11, should help reduce the phase angle uncertainty due to scan angle interpolation.

7. Summary

FP-11' polarization sensitivity testing was performed under ambient conditions for JPSS J1 VIIRS sensor. Analysis showed the following:

- Linear polarization sensitivity for bands M1 – M4 was observed to be higher than the specified limit (as high as ~6.4 % for M1, ~4.3 % for M2, ~3.0 % for M3, and ~4.3 % for M4).
- Differences in linear polarization sensitivity with HAM side are as high as ~1 %, where HAM side 1 is generally larger.
- Large detector-to-detector and scan angle differences were observed within bands (up to ~4 % in M1). This is likely the result of angle of incidence changes on the filter assembly.
- Comparisons between test configurations in general agreed (using the BVONIR polarizer sheet with and without the Sonoma shaping filter).
- Scan angle interpolation showed good agreement between the measurements in terms of polarization factor (less than 0.11 %). The measurements agreed to within about 4.5 degrees in terms of phase angle for bands I1 and M1 – M4; larger differences were observed for bands I2 and M5 – M7 (up to 22 degrees), but were confined to detectors and scan angles near phase

angle shifts from 0 to 2π. Such shifts generally occur where the polarization factor approaches zero.

Table 1: Specified maximum polarization sensitivity [5]

Band	Sensitivity [%]
I2, M1, M7	3
I1, M2, M3, M4, M5, M6	2.5

Table 2: Data used in the FP-11' test analysis using the BVONIR polarizer without the Sonoma filter. Data for which the source was unstable are highlighted in red.

Test type	UAID	Collects	Scan angle	Samples
Stray light – dark	4304478	1 – 25	-8	1625 – 1654
Stray light – lollipop	4304482	1 – 25	-8	1625 – 1654
Efficiency	4304484	1 – 25	-8	1625 – 1654
	4304517	1 – 25	-8	1625 – 1654
Polarization Sensitivity	4304488	1 – 25	-8	1625 – 1654
	4304491	1 – 25	-37	1065 – 1094
	4304494	1 – 25	-30	1455 – 1484
	4304499	1 – 25	-8	1625 – 1654
	4304503	1 – 25	-15	1230 – 1259
	4304507	1 – 25	4	1235 – 1264
	4304511	1 – 25	-8	1625 – 1654
	4304520	1 – 25	-8	1625 – 1654
	4304522	1 – 25	-37	1065 – 1094

Table 3: Data used in the FP-11' test analysis using the BVONIR polarizer with the Sonoma filter. Data for which the source was unstable are highlighted in red.

Test type	UAID	Collects	Scan angle	Samples
Stray light – dark	4304477	1 – 25	-8	1625 – 1654
Stray light – lollipop	4304481	1 – 25	-8	1625 – 1654
Efficiency	4304483	1 – 25	-8	1625 – 1654
	4304513	1 –25	-8	1625 – 1654
Polarization Sensitivity	4304486	1 – 25	-8	1625 – 1654
	4304490	1 – 25	-37	1065 – 1094
	4304493	1 – 25	-30	1455 – 1484
	4304497	1 – 25	-8	1625 – 1654
	4304501	1 – 25	-15	1230 – 1259
	4304505	1 – 25	4	1235 – 1264
	4304509	1 – 25	-8	1625 – 1654

Table 4: Maximum polarization factors (a_2) for HAM side 0

Band	Sensor	Scan Angle										
		-55	-45	-37	-30	-20	-15	-8	4	22	45	55
I1	SNPP	1.50	1.24	~	~	0.93	~	0.85	~	0.70	0.64	0.62
	J1	0.81	0.74	0.76	0.73	0.73	0.73	0.78	0.76	0.82	0.84	0.85
I2	SNPP	0.27	0.29	~	~	0.34	~	0.37	~	0.47	0.51	0.51
	J1	0.73	0.62	0.55	0.46	0.36	0.36	0.43	0.50	0.50	0.61	0.66
M1	SNPP	2.99	2.63	~	~	1.95	~	1.79	~	1.42	1.21	1.40
	J1	5.12	5.26	5.37	5.48	5.54	5.57	5.63	5.69	5.65	5.50	5.37
M2	SNPP	2.11	1.97	~	~	1.63	~	1.53	~	1.28	1.17	1.29
	J1	3.72	3.79	3.88	3.91	3.90	3.91	3.94	3.95	3.89	3.96	4.02
M3	SNPP	1.20	1.14	~	~	0.90	~	0.82	~	0.61	0.70	0.80
	J1	2.89	2.85	2.86	2.81	2.73	2.71	2.68	2.63	2.62	2.79	2.82
M4	SNPP	1.05	1.10	~	~	1.19	~	1.16	~	1.00	0.88	0.84
	J1	3.61	3.90	4.09	4.15	4.16	4.21	4.19	4.17	4.04	3.88	3.79
M5	SNPP	1.19	1.02	~	~	0.85	~	0.84	~	0.76	0.73	0.69
	J1	1.90	1.86	1.89	1.85	1.82	1.84	1.80	1.82	1.81	1.80	1.80
M6	SNPP	0.99	0.96	~	~	0.94	~	0.94	~	0.88	0.82	0.76
	J1	1.61	1.32	1.13	1.00	0.86	0.85	0.79	0.74	0.73	0.74	0.76
M7	SNPP	0.17	0.19	~	~	0.25	~	0.28	~	0.38	0.42	0.41
	J1	0.73	0.62	0.54	0.46	0.36	0.36	0.32	0.38	0.44	0.55	0.60

Table 5: Maximum polarization factors (a_2) for HAM side 1

Band	Sensor	Scan Angle										
		-55	-45	-37	-30	-20	-15	-8	4	22	45	55
I1	SNPP	0.86	0.76	~	~	0.62	~	0.59	~	0.54	0.58	0.61
	J1	0.86	0.90	0.94	0.94	0.94	0.97	0.95	0.97	1.00	1.03	1.03
I2	SNPP	0.49	0.45	~	~	0.47	~	0.51	~	0.56	0.56	0.55
	J1	1.19	0.92	0.76	0.62	0.50	0.49	0.48	0.50	0.53	0.58	0.61
M1	SNPP	3.14	2.73	~	~	2.01	~	1.83	~	1.45	1.23	1.39
	J1	5.57	5.73	5.88	6.01	6.17	6.17	6.20	6.31	6.41	6.16	5.95
M2	SNPP	2.25	2.05	~	~	1.65	~	1.54	~	1.28	1.17	1.30
	J1	4.08	4.08	4.15	4.19	4.18	4.20	4.23	4.25	4.19	4.34	4.45
M3	SNPP	1.45	1.31	~	~	0.96	~	0.85	~	0.62	0.71	0.81
	J1	2.92	2.86	2.86	2.82	2.76	2.76	2.75	2.74	2.85	3.06	3.09
M4	SNPP	1.59	1.52	~	~	1.37	~	1.30	~	1.02	0.86	0.82
	J1	4.03	4.20	4.33	4.34	4.32	4.34	4.31	4.28	4.15	3.99	3.90
M5	SNPP	0.81	0.74	~	~	0.70	~	0.69	~	0.61	0.59	0.57
	J1	2.10	2.17	2.20	2.18	2.13	2.13	2.13	2.07	2.02	1.99	1.97
M6	SNPP	1.29	1.14	~	~	0.96	~	0.92	~	0.81	0.75	0.70
	J1	1.03	0.92	0.89	0.86	0.86	0.90	0.91	0.94	0.95	0.95	0.94
M7	SNPP	0.52	0.47	~	~	0.43	~	0.44	~	0.48	0.47	0.45
	J1	1.18	0.92	0.74	0.61	0.48	0.47	0.44	0.45	0.47	0.52	0.56

Table 6: Maximum FP-11 scan angle interpolation differences from FP-11' measurements for a_2 [%]

Band	Scan Angle			
	-37	-30	-15	4
I1	0.07	0.05	0.06	0.06
I2	0.04	0.04	0.03	0.03
M1	0.07	0.09	0.10	0.10
M2	0.06	0.08	0.05	0.07
M3	0.05	0.06	0.04	0.05
M4	0.11	0.11	0.10	0.10
M5	0.06	0.05	0.06	0.05
M6	0.04	0.05	0.03	0.04
M7	0.03	0.04	0.03	0.04

Table 7: Maximum FP-11 scan angle interpolation differences from FP-11' measurements for δ_2 [degrees]

Band	Scan Angle			
	-37	-30	-15	4
I1	4.5	4.1	2.3	3.9
I2	4.4	6.6	21.5	17.7
M1	0.6	0.7	1.1	0.8
M2	0.8	0.8	0.6	0.6
M3	1.0	1.1	0.8	0.7
M4	1.6	1.3	1.3	1.4
M5	10.9	18.6	4.5	2.6
M6	5.3	11.6	11.2	15.9
M7	4.3	6.0	9.8	14.8

Figure 1: Schematic of the polarization sensitivity test setup

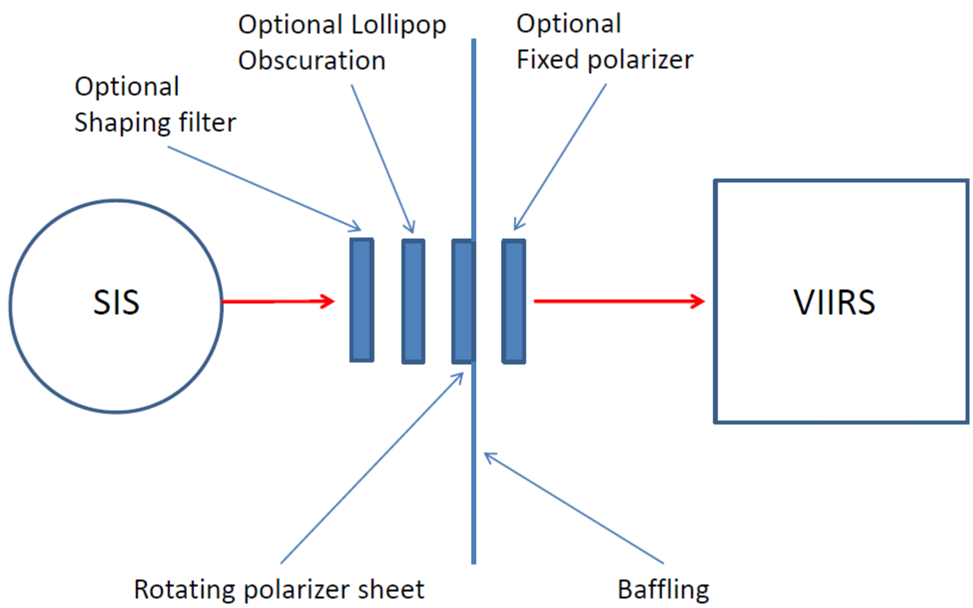

Figure 2: Schematic of the view of BVONIR polarizer from VIIRS with VIIRS coordinate system

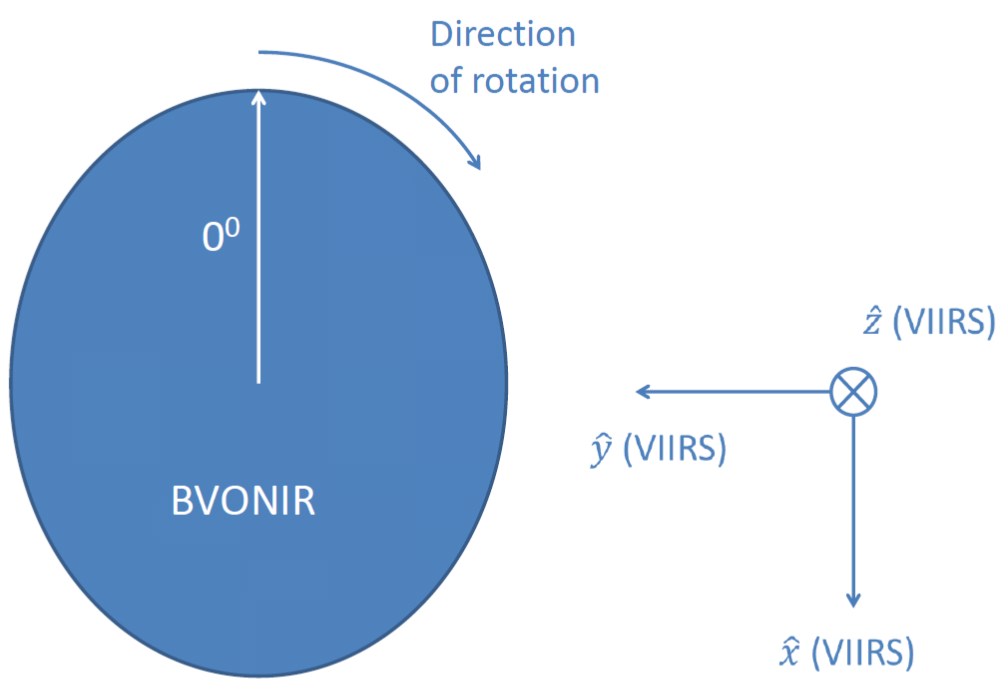

Figure 3: SIS-100-2 spectral profiles for two lamp configurations

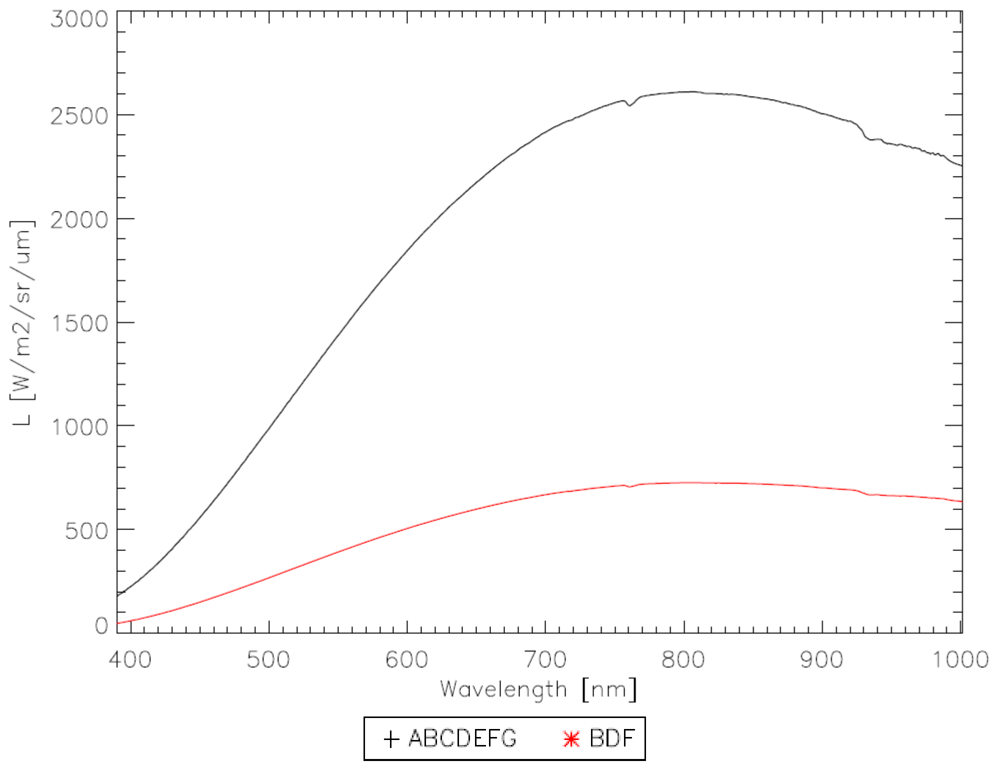

Figure 4: DoLP of the BVONIR polarizer

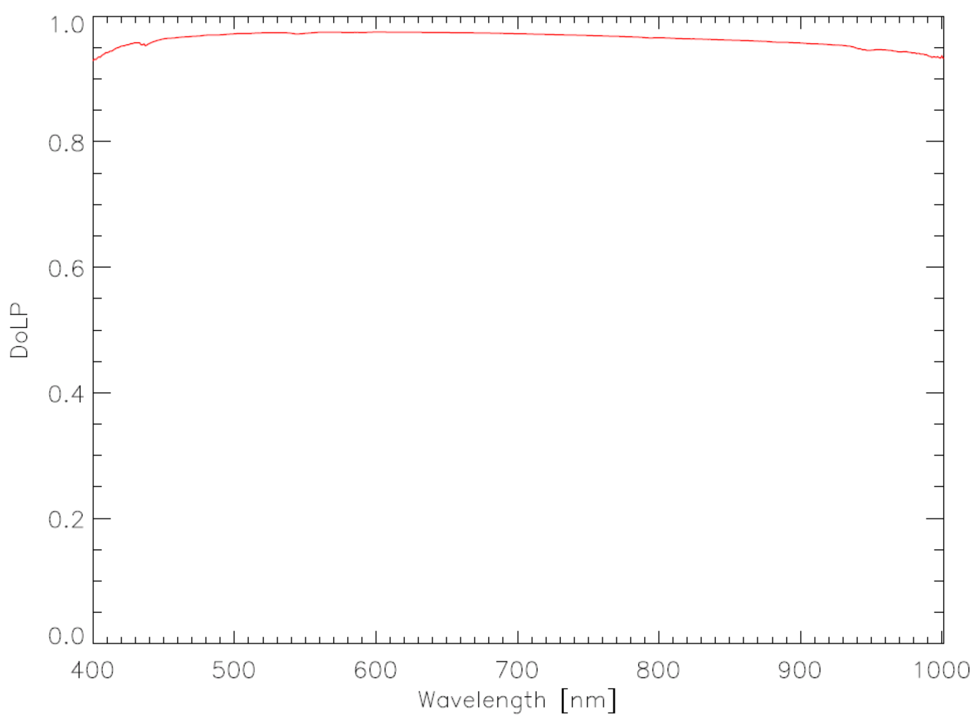

Figure 5: Transmittance of the Sonoma filter

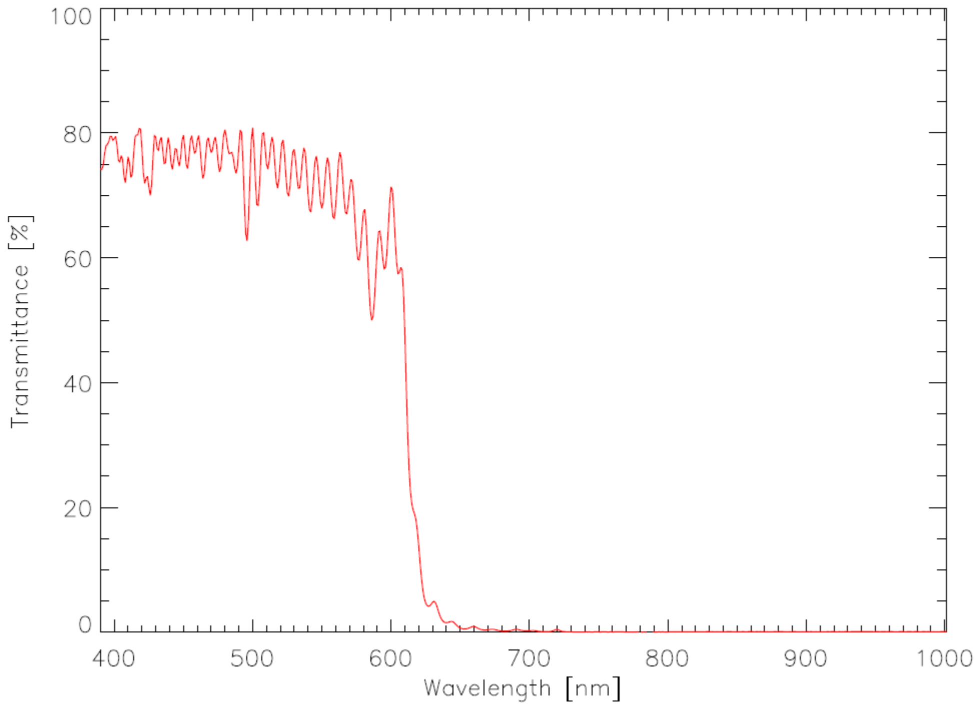

Figure 6: Input spectra for different test configurations

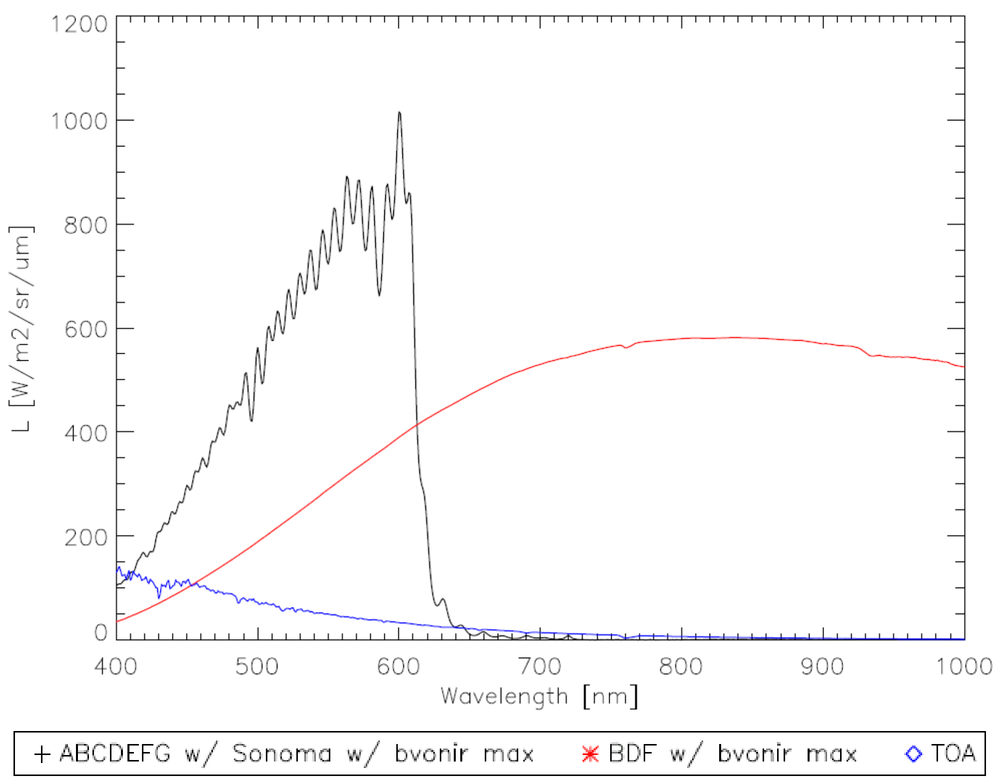

Figure 7: Stray light observed in M7 HAM 0 using the BVONIR polarizer (no Sonoma filter) with the source dark

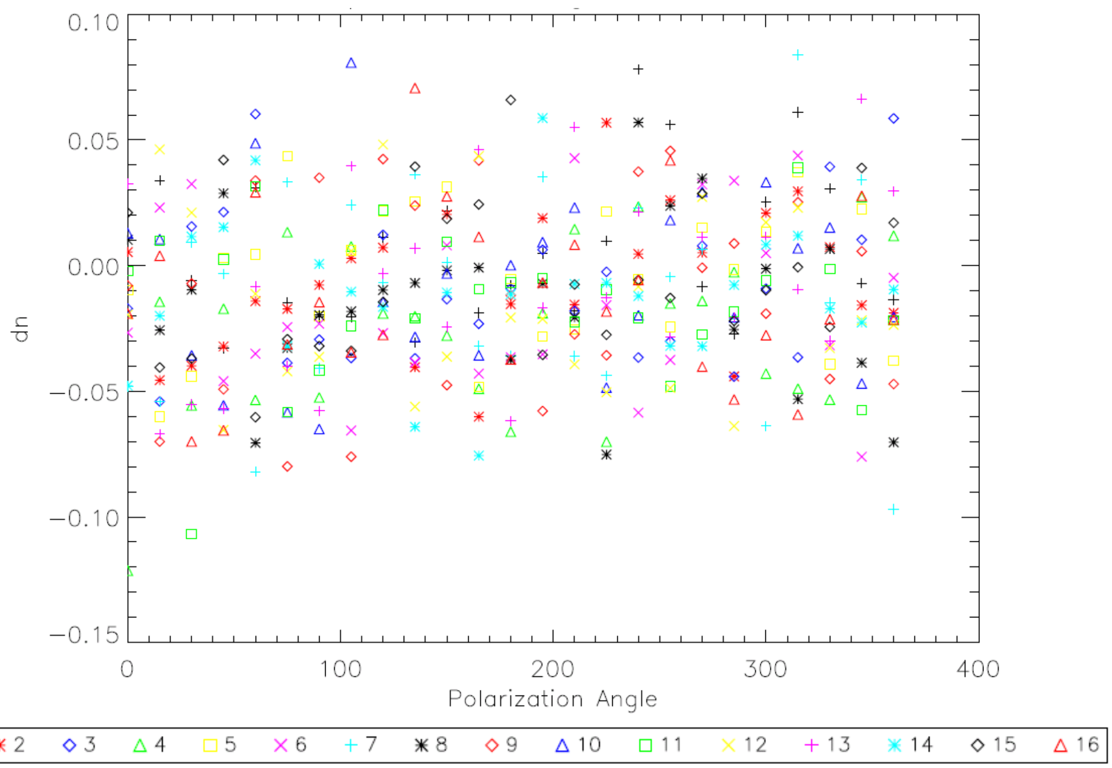

Figure 8: Stray light observed in M7 HAM A using the BVONIR polarizer (no Sonoma filter) with the lollipop obscuration

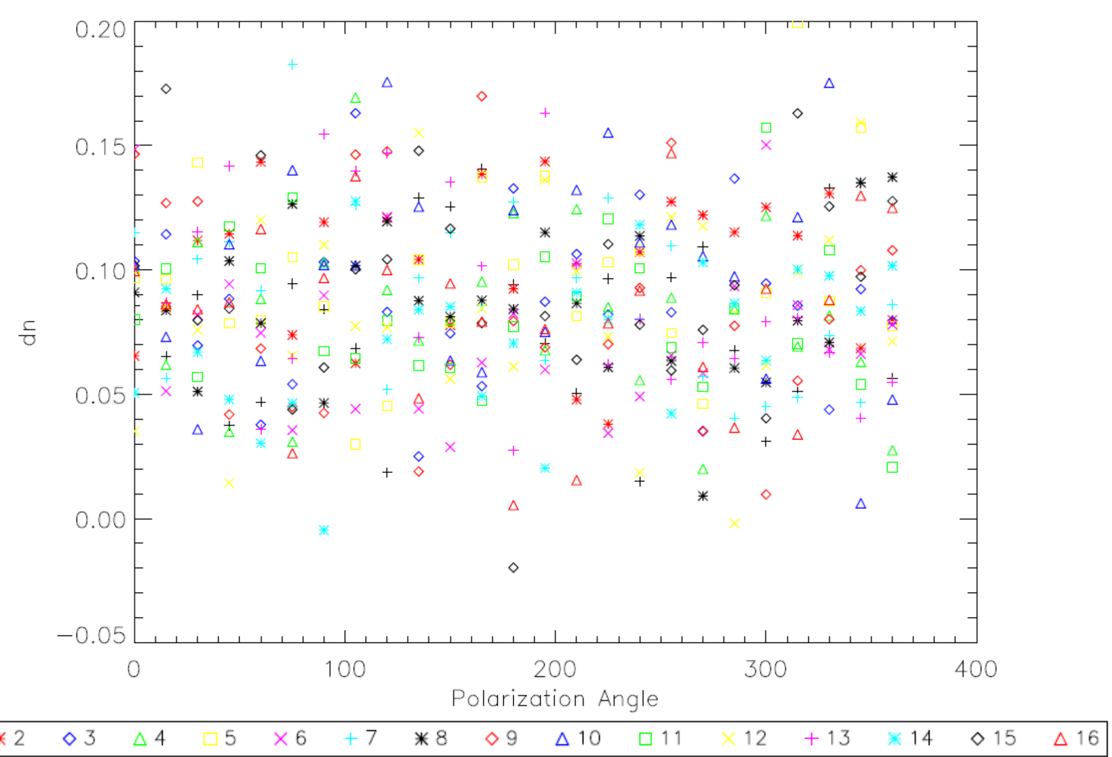

Figure 9: Band average FP-11' BVONIR polarizer efficiencies determined with and without the Sonoma filter, compared to FP-11 measurements

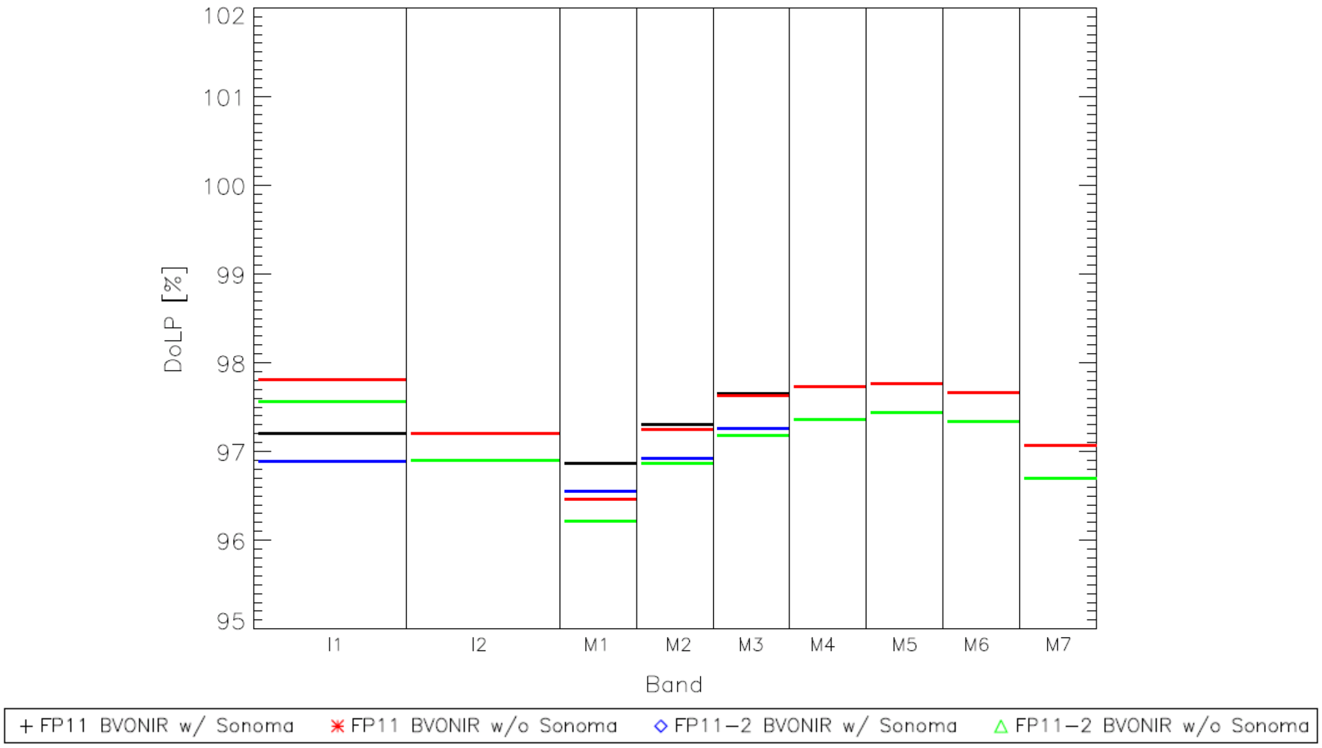

Figure 10: dn as a function of polarizer angle for M7 HAM 0 using BVONIR without the Sonoma filter (-8 degrees scan angle)

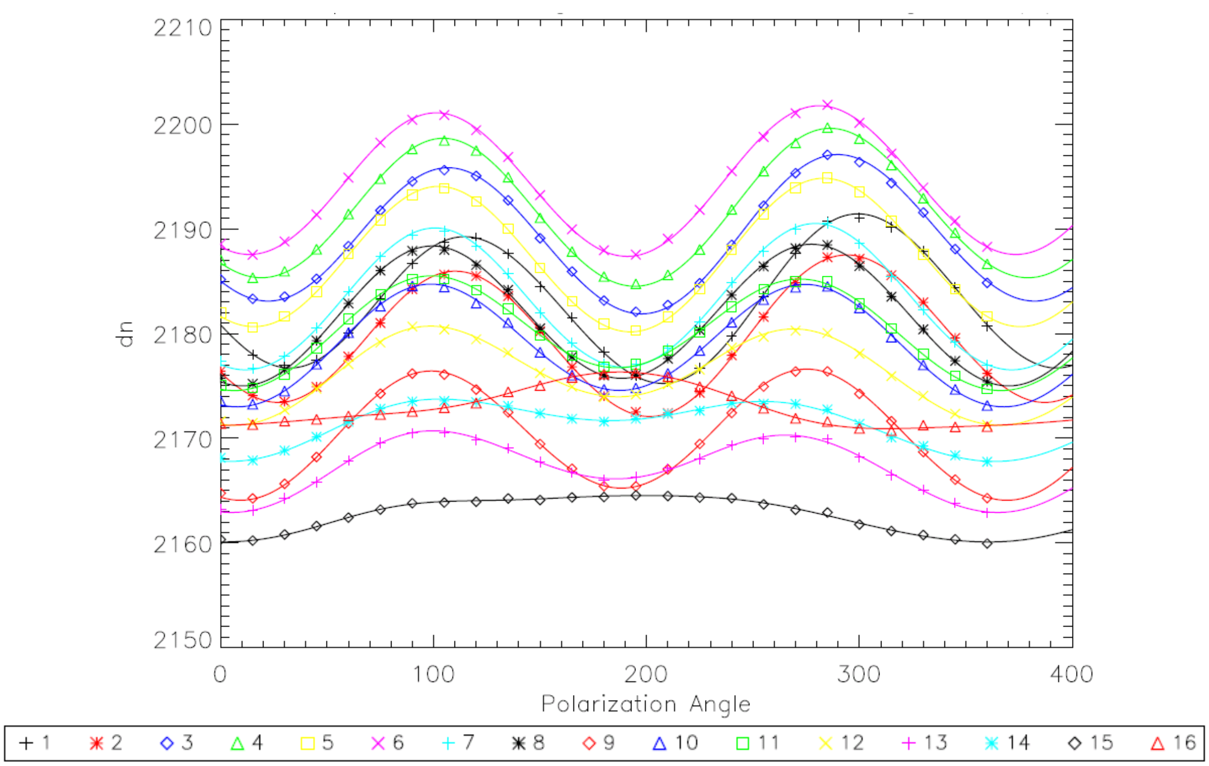

Figure 11: Fourier coefficients for M7 HAM 0 using BVONIR without the Sonoma filter

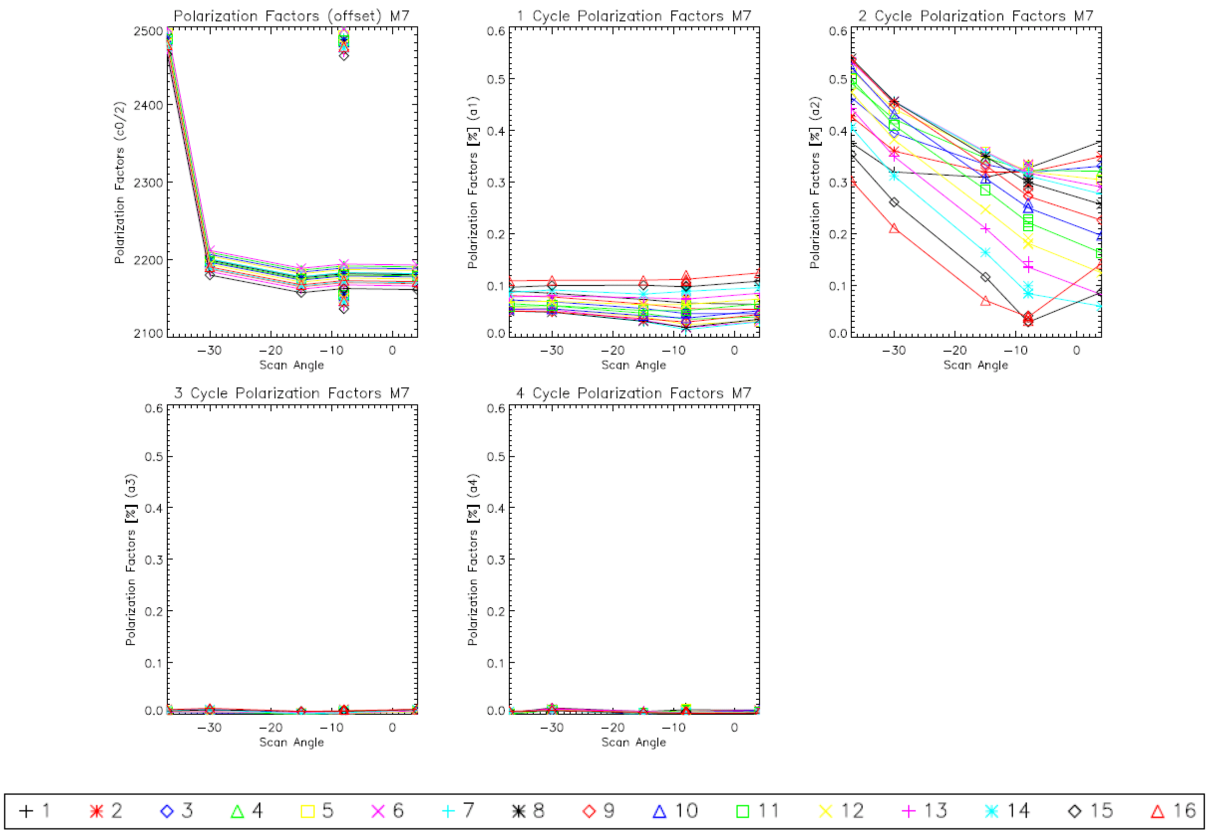

Figure 12: Polarization factor a_2 for M1, HAM side 0 in [%] across test configurations and repeated measurements

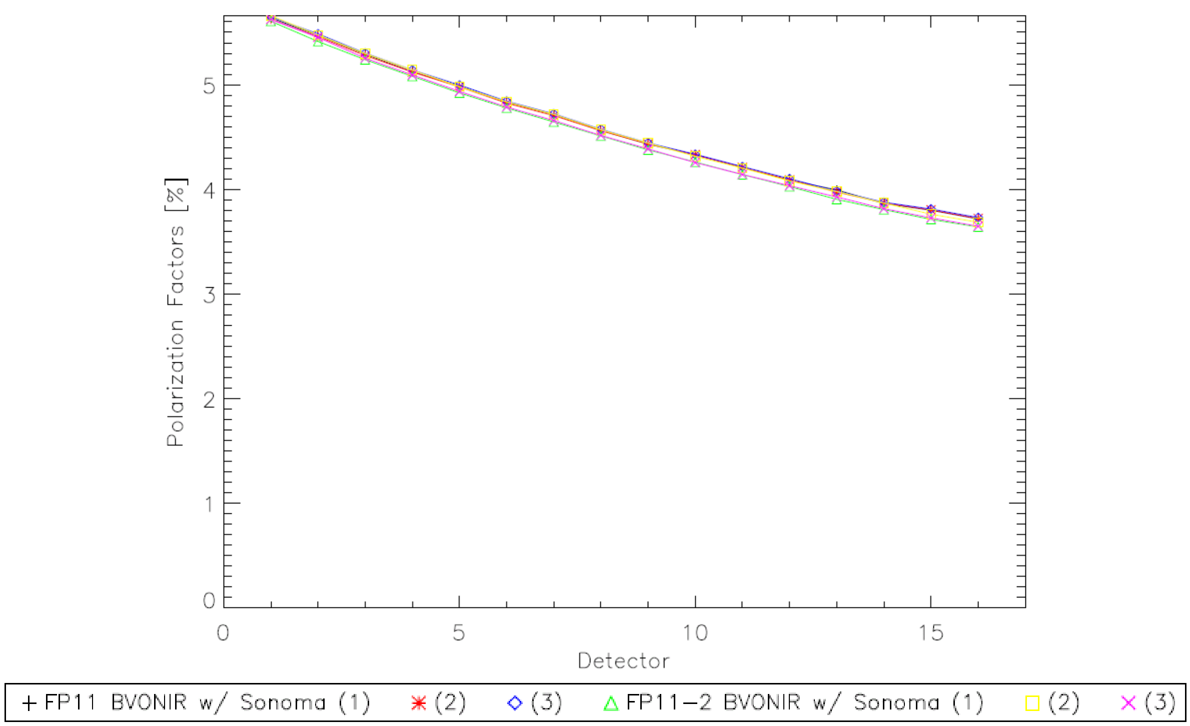

Figure 13: Polarization factor a_2 for HAM side 0 in [%] across scan angles

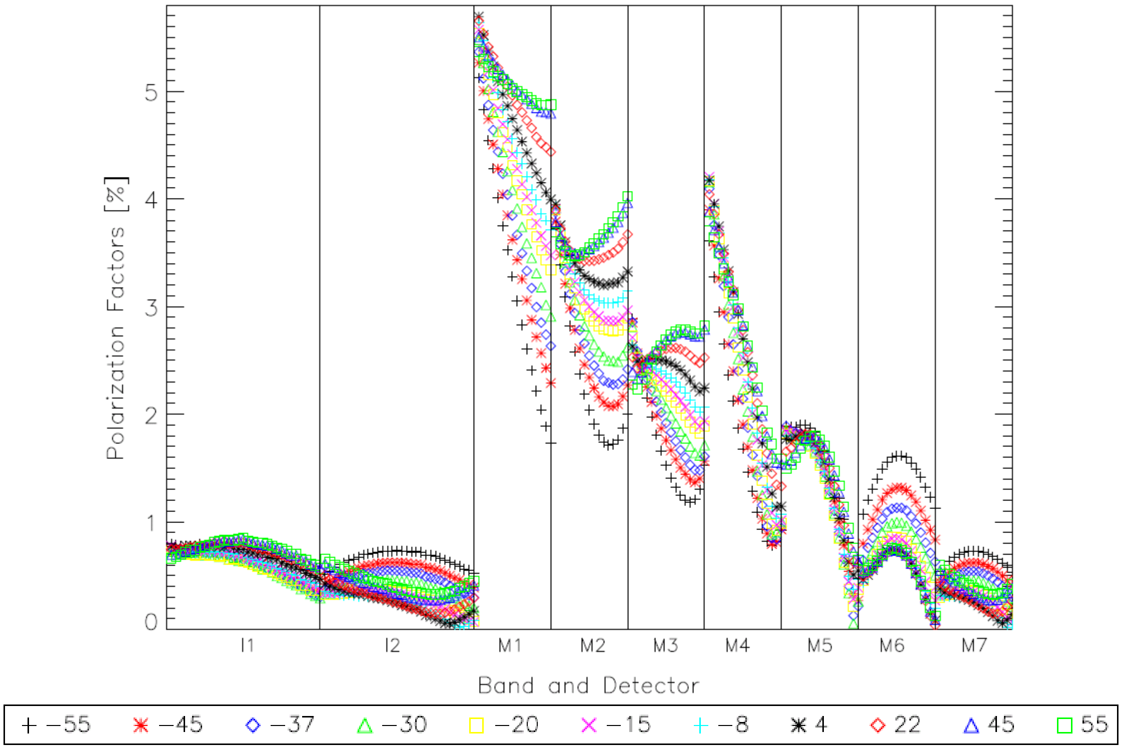

Figure 14: Polarization factor a_2 for HAM side 1 in [%] across scan angles

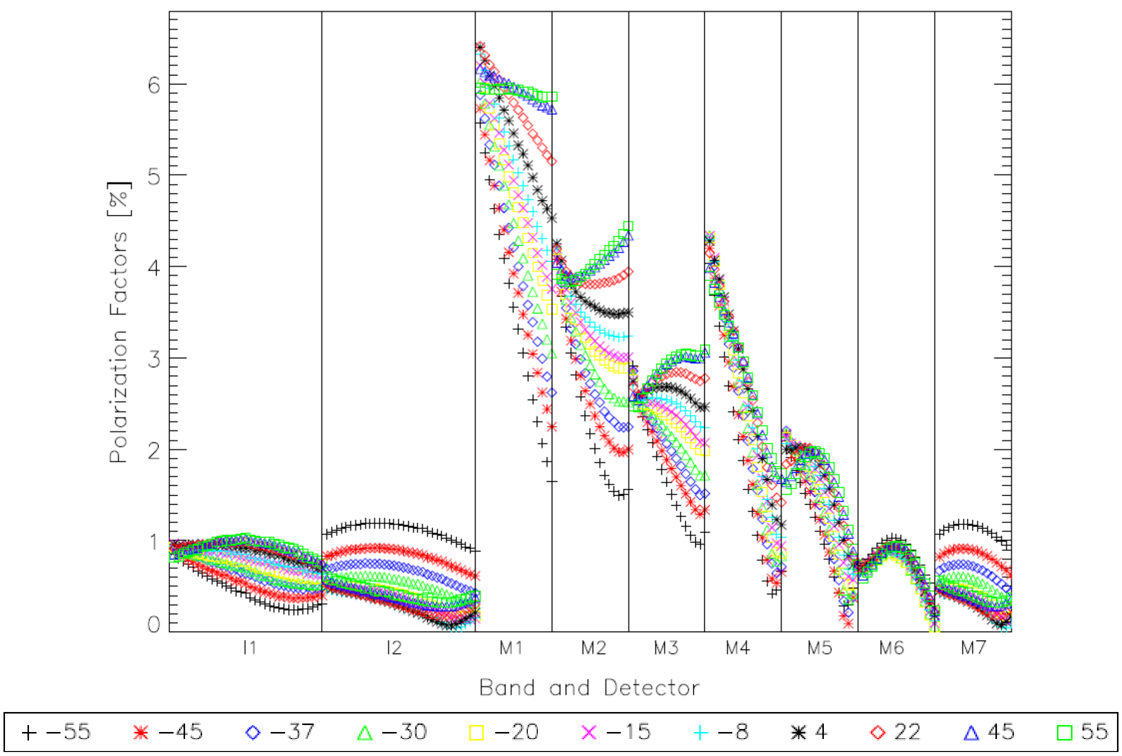

Figure 15: Polarization phase δ_2 for HAM side 0 in [%] across scan angles

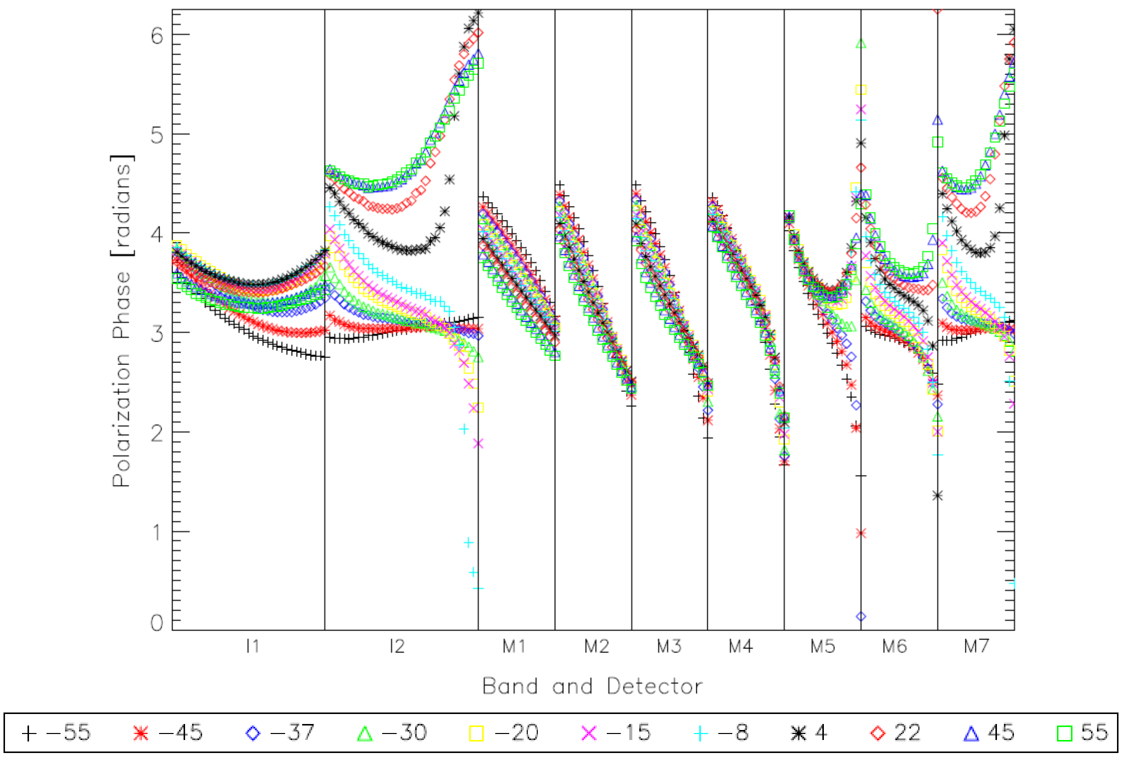

Figure 16: Polarization phase δ_2 for HAM side 1 in [%] across scan angles

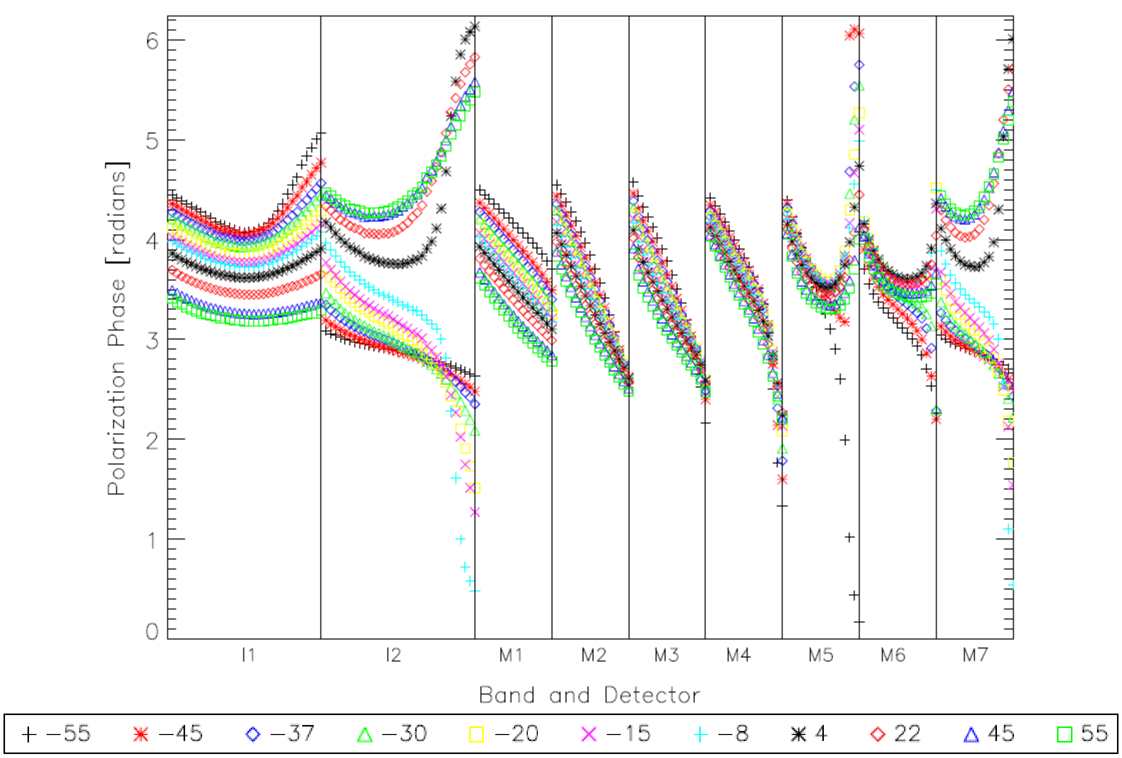

Figure 17: Phase angle for M5, HAM 0 versus scan angle

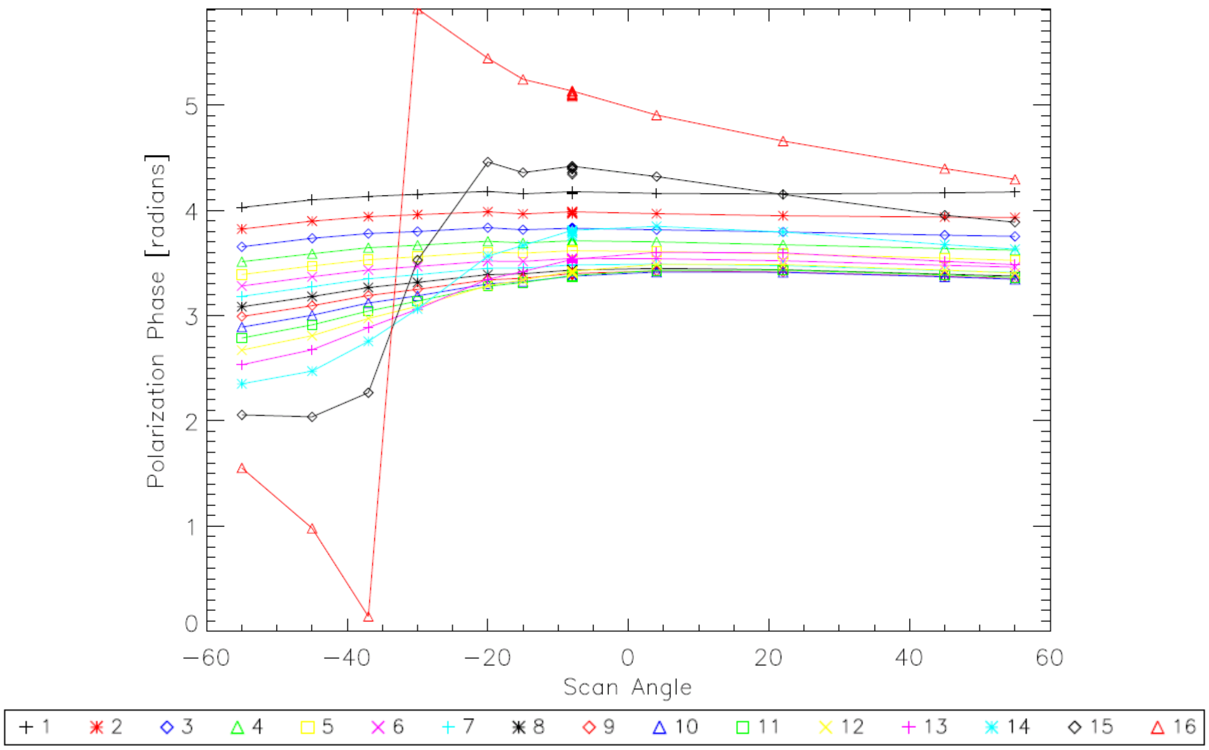

Figure 18: Phase angle for M7, HAM 0 versus scan angle